
Game-Based Learning With Direct Representation of Mathematics

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Abstract: Symbolic representations in mathematics (e.g., equations) are powerful and essential for more advanced mathematical thinking, but they cause major problems for K–8 learners. To engage mathematical reasoning without symbolic representations, BrainQuake has created diagrammatic mathematics puzzle games that provide an alternative, more learner-friendly interface to mathematical thinking and multistep problem solving. In this working paper, we first outline the design underlying BrainQuake’s puzzle games, and we provide preliminary evidence that they can be used effectively in classroom settings. The latter part of this paper outlines a randomized control study—currently in progress—examining how BrainQuake’s suite of puzzle games impacts students’ mathematics achievement and attitudes. The results of the randomized trial will be presented at the conference. This work on the whole provides a concrete illustration of how understanding of deeper cognitive processing can be leveraged to design learning games that effectively support students in reasoning in mathematics.

Introduction

Many K–12 students fail to realize their true mathematics potential, cutting them off from a wide variety of college majors and rewarding careers. This occurs at such a scale that it leads to a national skill shortage as well as limiting the individual student. BrainQuake designs and builds web and mobile learning puzzle games (for both classroom and home use) that solve three widespread and pervasive obstacles to the good mathematics learning that can improve students’ mathematical proficiency.

Students face many obstacles when learning mathematics in formal educational settings. The first is the *symbol barrier* (Devlin, 2011): namely, that mathematical symbols inherit a grammatical structure that supports mathematical thinking, but that learning to use these symbols has been known to cause major problems for K–8 learners (Devlin, 2011; Nunes, Carraher, & Schliemann, 1993). They also create a barrier that prevents individuals (particularly from more impoverished backgrounds) who lack the appropriate literacies from recognizing that they have the capacity for mathematical thinking, with the result that they do not make the effort that would lead to success. Keith Devlin has called this problem—which is one of language, not mathematics—the symbol barrier (Devlin, 2011).

The second obstacle is that there are no deep assessments that scale. Existing, scalable assessments mostly measure only what students have done, not how they did it. As a result, besides encouraging test prep (which unfairly favors students from more privileged backgrounds [College Board, 2013]), they miss the most valuable information: How did the student approach and think about the problem—even if they did not solve it?

Finally, the third obstacle is that students often carry a negative attitude toward mathematics. In some cases a definite math phobia (Tobias, 1995)—or a “fixed mindset” (Dweck, 2007) exists. Like the symbol barrier, negative attitudes and fixed mindsets are obstacles to good learning.

BrainQuake’s solution to these obstacles is to design products to provide an alternative, more learner-friendly interface to mathematical thinking and (multistep) problem solving, providing a means to break

the symbol barrier. This provides a direct solution to Obstacle 1, and solutions to Obstacles 2 and 3 follow automatically from the way we solve Problem 1 (Matlen, Atienza, & Cully, 2015). Because the game objects in BrainQuake’s products provide direct representations of mathematical concepts, players solve problems within the game itself. (They manipulate game objects instead of symbols on a page.) This enables the game to track solutions in detail and provide dashboard feedback to students, teachers, and parents, not just on performance but on possibly unrecognized mathematical proficiency, providing opportunities for targeted interventions. Being provided with information that they can do math (when suitably presented), people may start to develop a more positive attitude toward the subject (Dweck, 2007). Shute and Ventura (2013) call this kind of tracking *stealth assessment*, and they make powerful use of it in their science-learning game *Newton’s Playground*. The *Wuzzit Trouble (WT)* application (<http://wuzzittrouble.com>), available on iOS and Android platforms and in a browser version, is a puzzle game built on similar principles.

WT’s user interface (UI) is a representation of certain kinds of integer-arithmetic problems (integer partitions—the expression of a whole number as a sum of other whole numbers—and Diophantine equations) equivalent, but alternative, to the familiar symbolic algebra representation (trading in a static, spatial configuration of symbols for a dynamic interaction with a digital gears mechanism). The game was designed to develop number sense and general analytic problem-solving and optimization skills, while at the same time providing mathematically less-well-prepared players with practice of basic whole number skills.

Figure 1 shows just how big a difference a well-designed representation can make. In both cases, the “player” has to solve the problem, indeed by essentially the same sequence of steps. It is only the representations that are different. In addition to intimidating many students, the symbolic representation of the problem creates significant cognitive load, in large part because it is a static representation of an intrinsically dynamic process of solving a system of equations. In contrast, the representation on the left is dynamic. The player rotates either of the two small drive cogs (having four and six teeth, respectively) to rotate the large gearwheel. The object is to bring the keys (located at teeth 8 and 22) in line with the triangular marker at the top. (Simple puzzles have just one drive cog; more complicated puzzles have two, three, or four cogs.) Collecting all the keys in this manner releases the Wuzzit from the trap. A small cog may be wound up to rotate up to a set limit of times with a single player action. Maximum stars are obtained by releasing the Wuzzit with the fewest number of rotation actions, making optimization a key objective. What makes *WT* a powerful mathematics learning tool (i.e., not just arithmetic) is the complexity of the harder puzzles, for which optimizing the score requires sophisticated algorithmic reasoning. The fact that children as young as third grade can perform well on the easier puzzles, including children regarded as at remedial level (Pope & Mangram, 2015), confirms results from other research (e.g., Nunes et al., 1993) that when presented using a representation more efficient for learning, such reasoning is within the capacity of the average child.

Same Math, Different Representation

Collect the keys to free the Wuzzit



For maximum stars, use the least number of moves. For maximal points, collect the bonus items before you pick up the last key.

Solve the system of equations

$$\begin{aligned} 4x_1 + 6y_1 &= z_1 & (\text{mod } 65) \\ 4x_2 + 6y_2 &= z_2 - z_1 & (\text{mod } 65) \\ 4x_3 + 6y_3 &= z_3 - z_2 & (\text{mod } 65) \\ &\dots & \dots \\ 4x_n + 6y_n &= z_n - z_{n-1} & (\text{mod } 65) \end{aligned}$$

subject to the constraints

$$0 \leq x_i, y_i \leq 5, \quad x_i y_i = 0, \quad 1 \leq i \leq n$$

so that 8, 22, 32, 46 are members of the trace set

$$\begin{aligned} &\{4i \mid 1 \leq i \leq x_1\} \cup \{6i \mid 1 \leq i \leq y_1\} \cup \\ &\{z_1 + 4i \mid 1 \leq i \leq x_2\} \cup \{z_1 + 6i \mid 1 \leq i \leq y_2\} \cup \\ &\{z_2 + 4i \mid 1 \leq i \leq x_3\} \cup \{z_2 + 6i \mid 1 \leq i \leq y_3\} \cup \\ &\dots \cup \\ &\{z_{n-1} + 4i \mid 1 \leq i \leq x_n\} \cup \{z_{n-1} + 6i \mid 1 \leq i \leq y_n\} \end{aligned}$$

For maximum stars, solve with n minimal.

For maximal points, ensure that one of 8, 22 occurs in the final component of the trace set.

Figure 1. Wuzzit Trouble (left) and underlying mathematics (right).

Theoretical Framework

Though mathematics is often thought of as a collection of techniques for manipulating abstract symbols, it is in fact a powerful way of thinking about problems and issues in the world. Recognizing this fact, BrainQuake, unlike the vast majority of math-game developers in the market, places the emphasis of math education, particularly in middle and high schools, on developing mathematical thinking. Once students are able to think conceptually about mathematics, basic math skills are more easily acquired and far better retained (Kilpatrick, Swafford, & Findell, 2001).

According to this approach, in order to build truly successful mathematics learning games, developers must separate the activity of doing mathematics—a form of thinking—from its familiar representation in terms of symbolic expressions. To do so, educators and educational game developers must go beyond thinking of video games as a medium that delivers traditional pedagogy—a canvas on which to pour symbols—and instead see them as an entirely new medium to represent mathematical concepts. Concrete representations (such as representations used in some learning games, including *WT*) and the abstract symbolic representations in mathematics present a tradeoff in learning (Goldstone & Son, 2005; Koedinger, Alibali, & Nathan, 2008). Though students learn more efficiently when initially using

concrete representations (Resnick & Omanson, 1987), that knowledge is often less robust relative to when students learn from abstract representations (Kaminsky, Sloutsky, & Heckler, 2006).

BrainQuake's approach to mathematics education is based upon the five-interwoven-strands model recommended by the National Research Council's (NRC's) Year 2000 report *Adding It Up* (Kilpatrick et al., 2001). The five strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Devlin (2011) provides a lengthy, in-depth analysis of how good game design embodies all five strands. For successful education in this environment, the teacher has to understand both what is being taught (the math) and what is involved in learning math. Further, the student has to interact with both the math and the teacher. Most current pedagogic theory and practice is based on this model.

For classroom and class-related uses of BrainQuake products, BrainQuake completely reimagines this classroom learning framework by introducing the game as an element in pedagogic practice. The approach draws on two decades of thinking about how best to use digital games in formal education settings (e.g., Bransford & Schwartz, 1999). Specifically:

- Engage student preconceptions by drawing on the knowledge and experience that students bring to the classroom, but that are rarely activated in formal teaching and learning;
- Provide opportunities for students to experience discrepant events that allow them to build new knowledge and understanding on top of their existing models.

To make use of video games to provide experiential learning that meets the educational goals promoted by the NRC, BrainQuake relies on educational principles presented by Gee (2003). In Devlin (2011), BrainQuake's cofounder and chief scientist refined and extended Gee's general education principles for mathematics education. The most relevant lessons taken from those principles include:

- Interaction/Feedback: Games provide immediate feedback to player behavior, allowing the player to adjust and pursue new information in order to accomplish the goal.
- Risk Taking: Games reduce the consequences of failure, encouraging players to take risks, try new techniques, and learn from their mistakes.
- "Just in Time" and "On Demand": While people rarely learn effectively from information presented out of context, information presented when relevant is very likely to be retained. Just-in-time in-game instructions and hints can be presented to the player just when she approaches a new challenge.
- Performance Before Competence: In games, players can explore new, well-scaffolded tasks before they are fully competent.
- Personalized Learning: A real-time adaptive engine can present the player with learning challenges tuned to his current performance level.

Feasibility Study

The design of *WT* is based on well-grounded learning theory. However, its ultimate value for mathematics learning rests upon its ability to be used in context by practicing teachers and students. Toward this goal, a feasibility study of *Wuzzit Trouble* was conducted in 2015 with 205 students, six

teachers, two public schools. In the feasibility study, teachers were asked to use *WT* in their classrooms at least three times a week for 10 minutes a day during a two-week period. The study consisted of a mixed-methods (both qualitative and quantitative) approach that aimed to assess the feasibility of using *WT* in classroom contexts. The study was designed to explore the following questions:

- Does playing *WT* increase student learning and attitudes toward mathematics?
- What are teachers' impressions of *WT* and how do they implement *WT* in their classroom?

Teachers used *WT* double the minimum requirement for the study, averaging four days each week for 20 minutes each use. Teachers used a variety of implementation models for *WT*, including as a mathematical warm-up activity and as a translational activity integrated into the content of the mathematical lesson. Despite this variation, all participating teachers provided positive ratings toward questions of *WT*'s feasibility, for example, indicating that they found *WT* easy to use, helpful for supporting classroom lessons, and that they would use it again.

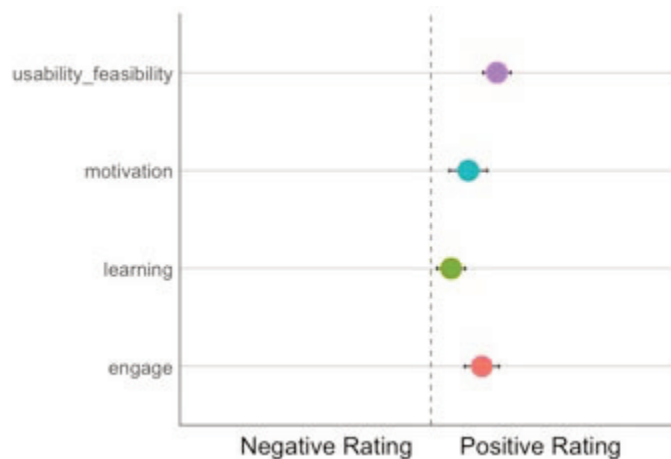


Figure 2. Mean student response for each survey subscale. Dotted line indicates neutral response. Error bars represent 95% confidence intervals.

Student ratings were assessed via a short Likert-response survey (see Figure 2). As can be seen in Figure 2, the students provided consistently positive findings across a range of subscales, including *WT*'s engagement (engage), usability/feasibility for classroom use (usability_feasibility), ability to improve motivation toward mathematics (motivation), and ability to support their mathematical learning (learning). All mean ratings were statistically different from a neutral rating in the positive direction ($p < .05$).

Overall, the 2015 feasibility study supports the conclusion that *WT* has strong potential for being an effective mathematics learning app that can be widely adopted for classroom use. Moreover, other independent studies of *WT* have shown similar findings, for instance, indicating that *WT* supports students' mathematical learning (Kiili, Devlin, Perttula, Tuomi, & Lindstedt, 2015; Pope & Mangram, 2015.)

Pilot Study

Based on the positive findings of the feasibility study described above, BrainQuake has since developed

two new games to assist students' mathematical reasoning. The new games are based in the same design theory as *WT*, but they target two additional areas of mathematics: algebraic and proportional reasoning. As with *WT*, the primary goal for the new puzzles is the development of (a) deep conceptual understanding (including number sense) and problem-solving capacity, (b) a positive disposition to mathematics, and (c) growth mindset.

Currently, we are conducting a randomized control study to explore the efficacy of the novel games. In the study, teachers are randomly assigned to either (a) use the games as a part of their mathematical lessons (treatment group), or (b) conduct mathematical lessons in the usual way, without the use of BrainQuake games (control group). This study is designed to address the following questions:

- Do BrainQuake products (henceforth referred to as the *BQ suite*) show promise for improving students' (a) mathematics achievement and (b) students' attitudes and beliefs toward mathematics, relative to a business-as-usual-control group?
- Does the BQ suite show promise for supporting teachers' pedagogical content knowledge in mathematics?
- How feasible is the BQ suite for classroom implementation?

Participants

Twenty-nine fifth- and sixth-grade teachers (15 control and 14 treatment) and approximately 812 students participated in the study. Participating schools came from rural, urban, and suburban school districts across California. Additional demographic information will be provided at the conference.

Measures

Multiple measures of both students and teacher are collected, including demographic and baseline information about students' mathematical proficiency. The primary outcomes in the randomized study are students' performance on (a) a content knowledge assessment—which includes both multiple-choice and open-ended questions—and serves to assess students' mathematical understanding, and b) a survey of students' mathematical attitudes and dispositions. Moreover, teachers' pedagogical content knowledge (PCK) for mathematics will be assessed using the Learning Mathematics for Teaching (LMT) assessment, which has been shown to have a large and statistically significant effect on students' mathematics achievement (Ball, Hill, & Bass, 2005). Finally, classroom observations, teacher interviews, and student focus groups will allow us to determine the extent to which BrainQuake games were feasible for classroom use, and how they were implemented in the context of everyday classrooms.

Data Analysis

Data collection will be completed by the end of January 2018. Analysis will be conducted and the findings will be presented at the 2018 CLS conference. Student and teacher surveys will be reverse coded and analyzed to determine whether usability, enjoyment, and feasibility questions differ from neutral responding—we will break down this analysis by socioeconomic status (SES) and gender. Usage data will help determine whether the adaptive engine delivers questions that are within students' ability and support student learning. For example, within a given difficulty level, we expect the number of moves until completion of puzzles to decrease across practice opportunities. Observational data will

be triangulated with weekly teacher logs and surveys in order to develop a narrative of how teachers used the game during classroom instruction and how teachers interpret and use teacher dashboard information.

Impact on student and teacher learning. Student and teacher pre- and posttest data will be analyzed with ANCOVA models, using gender, pretest scores, and SES as covariates. The effect of the treatment on student outcomes will be analyzed using hierarchical linear models to account for the nested structure of the design (students within teachers). The model will include students' scores on the posttest as the outcome (as measured by the MDTP or Attitudes survey), corresponding pretest measures, and fixed effect covariates for school (e.g., SES), teacher (e.g., LMT), and student levels (e.g., SES, gender), respectively.

Conclusion

The present work serves as a model example of how understanding of cognitive processes involved in learning mathematics can be leveraged to design learning games that effectively support students in mathematics. Preliminary findings thus far point to the conclusion that BrainQuake's games can serve as effective classroom learning tools that support mathematical reasoning, engagement, and competency. By the time of the conference, the findings from the pilot study will further inform the development of the BrainQuake games and how well-designed digital games can support student learning in schools.

References

- Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 29, 14–22.
- Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education*, 24, 61–100.
- College Board. (2013). *2013 college-bound seniors: Total group profile report*. Retrieved from <http://media.collegeboard.com/digitalServices/pdf/research/2013/TotalGroup-2013.pdf>
- Devlin, K. (2011). *Mathematics education for a new era: Video games as a medium for learning*. Boca Raton, FL: AK Peters/CRC Press.
- Dweck, C. S. (2007). *Mindset: The new psychology of success*. New York, NY: Ballantine.
- Gee, J. P. (2003). *What video games have to teach us about learning and literacy*. New York, NY: Palgrave Macmillan.
- Goldstone, R. L., & Son, J. Y. (2005). The transfer of scientific principles using concrete and idealized simulations. *Journal of the Learning Sciences*, 14(1), 69–110.
- Kaminsky, J. A., Sloutsky, V. M., & Heckler, A. F. (2006). Effects of concreteness on representation: An explanation for differential transfer. In R. Sun & N. Miyake (Eds.), *Proceedings of the XXVIII Annual Conference of the Cognitive Science Society*. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.590.9308&rep=rep1&type=pdf>

Kiili, K., Devlin, K., Perttula, A., Tuomi, P., & Lindstedt, A. (2015). Using video games to combine learning and assessment in mathematics education. *International Journal of Serious Games*, 2(4), 37–55.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

Koedinger, K. R., Alibali, M. W., & Nathan, M. J. (2008). Trade-offs between grounded and abstract representations: Evidence from algebra problem solving. *Cognitive Science*, 32, 366–397.

Matlen, B., Atienza, S., & Cully, K. F. (2015). *Wuzzit Trouble feasibility report* [Part of IES SBIR Phase I final report]. San Francisco, CA: WestEd.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics* (Learning in Doing: Social, Cognitive and Computational Perspectives). Cambridge, England: Cambridge University Press.

Pope, H., & Mangram, C. (2015). *Wuzzit Trouble: The influence of a digital math game on student number sense*. *International Journal of Serious Games*, 2(4), 5–22.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (pp. 41–95). Hillsdale, NJ: Lawrence Erlbaum.

Shute, V., & Ventura, M. (2013). *Stealth assessment: Measuring and supporting learning in video games*. Cambridge, MA: The MIT Press.

Tobias, S. (1995). *Overcoming math anxiety*. New York, NY: W. W. Norton.